Voltage Oriented Control of a Three Phase Rectifier.

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I. Introduction

This summarizes the steps to design a Voltage Oriented Controller (VOC) for a Three-Phase VSI working as a rectifier [1].

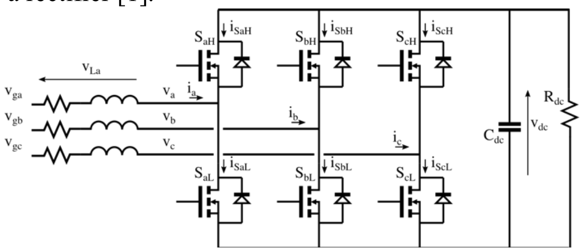

Figure 1. Three Phase Rectifier.

II. Voltage and currents references

The references in figure 1 have been considered. The following equations to model the system can then be written. A three phase, balanced grid voltage system, *abc*, is assumed, and therefore:

$$\left.\begin{aligned} v_a &= V_m \cdot \cos(\omega t) \\ v_b &= V_m \cdot \cos\left(\omega t - \tfrac{2\cdot\pi}{3}\right) \\ v_c &= V_m \cdot \cos\left(\omega t - \tfrac{4\cdot\pi}{3}\right) \end{aligned}\right\} \tag{1}$$

where V_m is the phase-to-neutral peak voltage, and it is assumed that voltage phase *a*, v_{ga}, is the reference for the phases. Also, ω is the grid angular frequency that can also be expressed by:

$$\omega = 2\pi f \tag{2}$$

where f is the line frequency.

Then the grid currents can be defined as:

$$\left.\begin{aligned} i_a &= I_m \cdot \cos(\omega t + \varphi) \\ i_b &= I_m \cdot \cos\left(\omega t + \varphi - \tfrac{2\cdot\pi}{3}\right) \\ i_c &= I_m \cdot \cos\left(\omega t + \varphi - \tfrac{4\cdot\pi}{3}\right) \end{aligned}\right\} \tag{3}$$

being I_m the peak phase current value, and φ the phase shift between current and voltage in phase *a*.

Assuming 3 wires, then:

$$i_a + i_b + i_c = 0 \tag{4}$$

III. Rectifier model and PWM Scheme

For each branch, the S function can be defined:

$$S_i = \begin{cases} 1 \rightarrow S_{iH} ON \wedge S_{iL} OFF \\ 0 \rightarrow S_{iH} OFF \wedge S_{iL} ON \end{cases}, \forall i \in [a, b, c] \tag{5}$$

Considering Fig. 1, then:

$$\left.\begin{aligned} v_{ga} &= R \cdot i_a + L \cdot \frac{di_a}{dt} + v_a \\ v_{gb} &= R \cdot i_b + L \cdot \frac{di_b}{dt} + v_b \\ v_{gc} &= R \cdot i_c + L \cdot \frac{di_c}{dt} + v_c \end{aligned}\right\} \tag{6}$$

The current node also gives a condition:

$$C\frac{dv_{dc}}{dt} = S_a \cdot i_a + S_b \cdot i_b + S_c \cdot i_c - i_{Load} \tag{7}$$

These equations can be expressed as:

$$\begin{bmatrix} v_{ga} \\ v_{gb} \\ v_{gc} \end{bmatrix} = R \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + L\frac{d}{dt}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{8}$$

$$C\frac{dv_{dc}}{dt} = [S_a \; S_b \; S_c]\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - i_{Load} \tag{9}$$

Or, alternatively:

$$\boldsymbol{v_{gabc}} = R \cdot \boldsymbol{i_{abc}} + L\frac{d}{dt}\boldsymbol{i_{abc}} + \boldsymbol{v_{abc}} \tag{10}$$

$$C\frac{dv_{dc}}{dt} = \boldsymbol{S^t_{abc}} \cdot \boldsymbol{i_{abc}} - i_{Load} \tag{11}$$

IV. Transformations

So far, all the waveforms are AC waveforms. The following transformations are carried out in order to simplify the control [2].

The three-phase *abc* system is initially transformed into a 2-D orthogonal rotating system, called αβ representation. The three-phase voltages and currents can be represented by means of rotating vectors in a plane, with a 120º shift between phases, as in figure 1.

The idea underneath this abc/αβ transformation is to represent the three-phase vector system into a single vector in an orthogonal stationary reference frame, i.e. the αβ axes system. If the stationary reference frame is chosen wisely, then the components of the single vector in this reference frame can provide a lot of information on the operation of the system.

Firstly, the αβ transform is obtained:

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \frac{2}{3} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{3} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{12}$$

Obviously,

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \tag{13}$$

This is called the amplitude invariant Clarke Transformation.

For instance, from the three-phase grid voltage system, an *abc*-αβ transformation can be carried out, with an arbitrary location of the αβ axes. The third component, γ, will be zero if the system is balanced. Notice then that the α and β values are orthogonal components of a single vector, that has the information of the three-voltage grid system. There is a significant reduction in the complexity of the analysis of the voltage systems here; something analogous to the single-phase equivalent used in basic analysis of circuits, but now including the possibility of accounting for unbalances, transients, etc.

What is interesting in this transformation is not only the resulting αβ projections of the voltage and current resulting vectors, but also the relationships among these single vectors. If the transformation is also carried to the three-phase grid currents, the same phase shift φ, as defined in (3) is held between the resulting current and voltage transformed single vectors. The key issue is that the angle between the resulting current and voltage single vectors is related with the ratio of active and reactive power demanded from the grid.

This is also very useful in machine control, as it transforms phase currents into field-oriented magnitudes; if the original currents are the ones in a motor, then these currents are the torque and flux producing currents. They are still AC waveforms, but in a stationary reference framework, thus revealing the position of the rotor. But in the present case, there's not a machine to control. However, the transformation is still valid; instead of the position of the rotor, the grid angle can be interpreted, referring to the voltage coordinates (hence its name, voltage oriented control). Instead of a torque and flux producing currents, active and reactive components of the currents can be derived.

Assuming a three-phase balanced system, then the aforementioned equations convert into:

$$\begin{bmatrix} v_{g\alpha} \\ v_{g\beta} \end{bmatrix} = R \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + L\frac{d}{dt}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \tag{14}$$

$$C\frac{dv_{dc}}{dt} = \frac{3}{2}\left(S_\alpha \cdot i_\alpha + S_\beta \cdot i_\beta\right) - i_{Load} \tag{15}$$

Still, these αβ magnitudes are AC waveforms, as previously mentioned. The following transformation converts these AC magnitudes into DC values.

$$\begin{bmatrix} d \\ q \\ z \end{bmatrix} = \begin{bmatrix} cos(\theta) & sin(\theta) & 0 \\ -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \tag{16}$$

where θ is the phase angle of the αβ grid voltage vector (i.e. the grid angle). The selection of the θ reference is critical, as now the resulting current angle is directly the power factor (assuming sine waveforms and a balance system)

This second transformation is called the Park transformation. By carrying out both transformations, then the original AC three-phase waveforms are converted into two DC magnitudes, d and q, which are much easier to control. In the case of a rectifier, the d component of the current will control the active power demanded from the grid, while the q component will fix the reactive power. This enables then for full power factor control of the input power.

V. Measurements

In order to carry out an adequate voltage oriented control of the inverter, the following magnitudes need to be measured, either directly or indirectly:

- Grid voltage waveforms, v_{ga}, v_{gb} and v_{gc} (in order to calculate the dq components).
- Grid current waveforms, i_{ga}, i_{gb} and i_{gb} (in order to also get the dq components).
- DC link voltage, v_{dc}, that is the parameter to be controlled.
- Grid angle, θ, that is required to perform the abc/αβ/dq transformations, and it also provides the power factor control capability.

This last parameter will be discussed in the next section. As it's calculated indirectly from the grid voltages. For the rest of the magnitudes, voltage/current transducers will be used. Obviously, all of them must follow the required signal conditioning stage, that includes adapting levels to the ADC limit values, and the anti-aliasing filter (AAF) [3].

VI. Implementation of the transformations

In order to implement the Clarke-Park transformation on the measured currents, the measurement of the angle of the phase voltage is required. There are several ways to measure this angle; for input grid voltage, one of the most usual ways to do it is by using a Phase-Locked-Loop (PLL) block.

The following block diagram shows a typical PLL scheme, that can be relatively easy to implement. Still, this kind of control blocks can be very sensitive to noise, harmonics, etc.

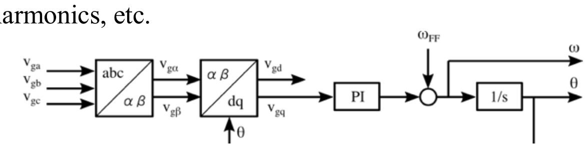

Figure 2. PLL control.

The key is to select adequate values for the PI controller parameters. As a first guess, the values for K_{IPLL} and K_{PPLL} can be chosen as:

The PI regulator is given by:

$$PI_{PLL}(s) = K_{PPLL} \cdot \left(1 + \frac{K_{IPLL}}{s}\right) \tag{17}$$

$$K_{PPLL} = \frac{2 \cdot BW_{PLL}}{V_m} \tag{18}$$

$$K_{IPLL} = \frac{{BW_{PLL}}^2}{V_m} \tag{19}$$

where BW_{PLL} is the bandwidth of the PLL. For reference, a value of BW_{PLL}=10Hz can be initially targeted [6].

But once the PLL is working, it must be noticed that the grid voltage will have only a direct component, v_{gd}, that is to say:

$$v_{gq} = 0 \tag{20}$$

This means that the grid current component in the d axis provides a current in phase with the grid voltage, e.g. a current that provides only active power. On the other hand, the q component of the current provides reactive power to the system.

VII. Control of the VSC rectifier

The control scheme can be understood as in Fig. 3. The grid voltage and current magnitudes are transformed into dq coordinates, also obtaining the grid voltage phase and frequency information from the PLL block. These parameters, together with the v_{dc} voltage are used as measured magnitude in the control stage.

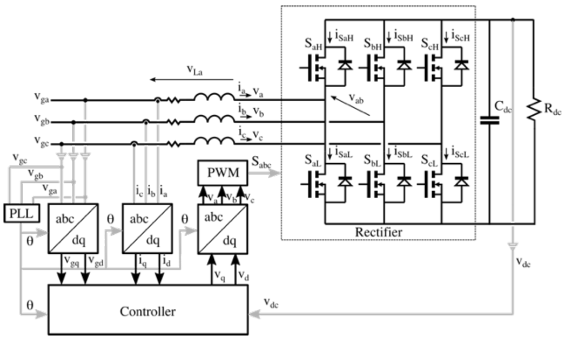

Figure 3. Control block diagram

The idea is to control the output DC link voltage, v_{dc}, by means of a voltage control loop. The output of this external loop provides the reference for the grid i_d direct current. Indeed, this i_d current is in phase with the grid voltage, as stated below, and takes into account the active power entering to the system. Thus, if the power demand at the load suddenly increases, the DC link voltage will decrease, and the voltage control must increase the reference for the d current i_d*, so as to provide more active power to the load and, consequently, increase back to the nominal value the DC link voltage level.

The reference for the i_q* component is defined as a function of the desired power factor at the input of the rectifier. In case unity power factor is required, this reference can be set to 0.

This way, the reference for the grid d and q coordinates is now fixed; two internal current control loops can be therefore schematized as in figure 4 [1][2][5].

The following define some hints to tune the PI controllers of the voltage and current control loops, but a standard approach for a cascaded control design can be followed.

a) DC link Voltage control Loop

The voltage control loop follows the outline shown in figure 5.

The transfer function is defined as:

$$G_v(s) = \frac{v_{dc}}{i_d^*} \tag{21}$$

In order to get this transfer function, the active power is investigated. On the grid side:

$$P = \frac{3}{2}\left(v_{gd} \cdot i_d + v_{gq} \cdot i_q\right) = \frac{3}{2}\left(v_{gd} \cdot i_d\right) \tag{22}$$

On the other side, at the DC link:

$$P = \frac{dE_C}{dt} + P_{LOAD} = \frac{1}{2} C \frac{d{v_{dc}}^2}{dt} + \frac{{v_{dc}}^2}{R_{dc}} \tag{23}$$

Henceforth:

$$\frac{3}{2}\left(v_{gd} \cdot i_d\right) = \frac{1}{2} C \frac{d{v_{dc}}^2}{dt} + \frac{{v_{dc}}^2}{R_{dc}} \tag{24}$$

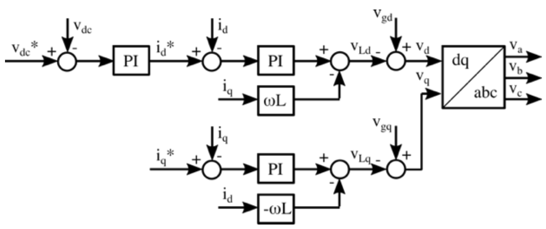

Figure 4. Decoupled control diagram.

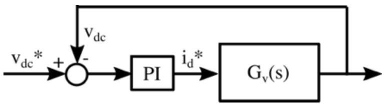

Figure 5. DC link voltage control loop scheme.

Considering each variable x(t) to be equal to the steady state value, X, and the perturbation, $\hat{x}$

$$x = x(t) = X + \hat{x} \tag{25}$$

Then, from (24):

$$G_v(s) = \frac{3}{2}\frac{v_d}{v_{dc}}\frac{R_{dc}}{1+R_{dc} \cdot C_{dc} \cdot s} \tag{26}$$

The block diagram then can be defined as the one in figure 6.

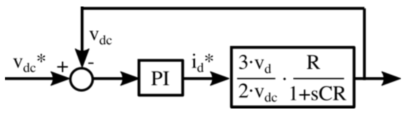

Figure 6. DC link voltage control loop with DC link plant.

The PI regulator is given by:

$$PI_v(s) = K_{PV} \cdot \left(1 + \frac{K_{IV}}{s}\right) \tag{27}$$

A zero-pole cancellation scheme can be implemented in the control, following the usual procedure [4], yielding to:

$$K_{IV} = \frac{1}{RC} \tag{28}$$

$$K_{PV} = 2\pi \cdot BW_v \cdot C \cdot \frac{3 \cdot v_d}{2 \cdot v_{dc}} \tag{29}$$

where BW_v is the target bandwidth of the voltage controller. However, it must be noticed that there is a manifold of possible controllers and different control tuning strategies for each of these.

b) Current control loops

In a similar manner, the control loops can be tuned assuming the plant schematized in figure 7 [5].

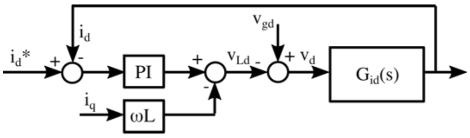

Figure 7. Current control loop (d-component)

After considering the decoupling terms in the diagram, the control loops for the d- and q- coordinates of the filter current can be schematized as in figure 8.

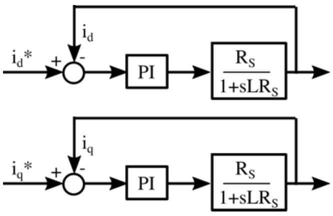

Figure 8. Current control loop with filter plant.

The poles cancellation scheme followed for the DC link control might not be very effective, since the value of the parasitic series resistor of the filter, R_S, is by definition small, and might change significantly with operating parameters such as temperature or current levels. Instead, the filter can be assumed as a pure integrator for the PI tuning. A simple proportional controller is not acceptable, though, since the small resistor present in the real device would provide errors.

For a PI defined analogously as in the voltage loop case:

$$PI_i(s) = K_{PI} \cdot \left(1 + \frac{K_{II}}{s}\right) \tag{30}$$

Then the closed loop response, considering a pure inductor L, is given by:

$$G_{CLi} = \frac{1+\frac{1}{K_{II}}\cdot s}{1+\frac{1}{K_{II}}\cdot s+\frac{L}{K_{II}\cdot K_{PI}}\cdot s^2} \tag{31}$$

Identifying terms with the second order expression as a function of the natural frequency, f_n, and the damping factor, ζ,

$$G_{CLi} = \frac{1+\frac{1}{K_{II}}\cdot s}{1+\frac{1}{K_{II}}\cdot s+\frac{L}{K_{II}\cdot K_{PI}}\cdot s^2} = \frac{1+\frac{2\cdot\zeta}{2\cdot\pi\cdot fn}\cdot s}{1+\frac{2\cdot\zeta}{2\cdot\pi\cdot fn}\cdot s+\left(\frac{1}{2\cdot\pi\cdot fn}\right)^2\cdot s^2} \tag{32}$$

then the following relations that define the PI controller can be derived,

$$K_{PI} = 4\cdot\pi\cdot BW_i\cdot L\cdot\zeta \tag{33}$$

$$K_{II} = \frac{2\cdot\pi\cdot BW_i}{\zeta} \tag{34}$$

where it has been considered that the bandwidth of the current control loop, BW_i, is equal to the natural frequency, f_n.

VIII. Stability issues

The system stability must be studied. Figure 9 represents the closed loop cascaded control scheme. The following expression can be derived for both the open loop gain:

$$G_{vOL}(s) = PI_V(s)\cdot\frac{PI_I(s)\cdot\frac{R_S}{1+s\cdot L\cdot R_S}}{1+PI_I(s)\cdot\frac{R_S}{1+s\cdot L\cdot R_S}}\cdot\frac{3}{2}\frac{v_d}{v_{dc}}\frac{R_{dc}}{1+R_{dc}\cdot C_{dc}\cdot s} \tag{35}$$

The closed loop gain, then, can be defined as:

$$G_{vCL}(s) = PI_V(s)\cdot\frac{PI_V(s)\cdot G_{vOL}(s)}{1+PI_V(s)\cdot G_{vOL}(s)} \tag{36}$$

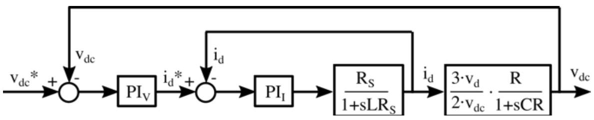

Figure 9. DC link voltage control loop with DC link plant.

The stability of the system can be assessed by looking at the Bode plot of the open loop gain, $G_{vOL}(s)$.

Figure 10 shows the open loop gain of the system, and a phase margin of around 75º (greater than a estimated safe value of 60º) is achieved, and hence the system is stable in the nominal operating conditions.

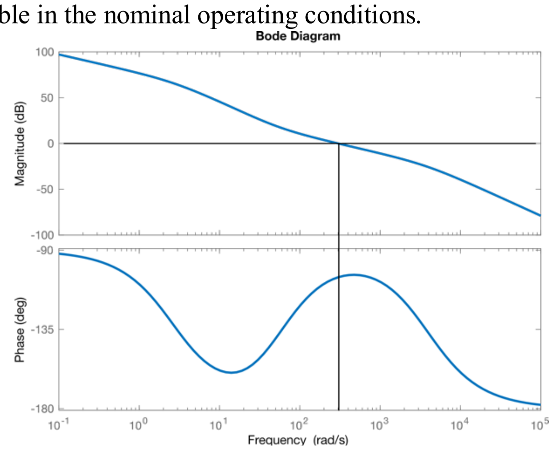

Figure 10. DC link voltage control loop with DC link plant.

IX. Conclusions

The present report has analyzed and provided a step-by step design of a three-phase VSI operating as a controller rectifier. The system is inherently bidirectional. Therefore, the system will work exactly the same as an inverter, choosing a negative i_d* reference as defined in the previous sections.

It must be noticed that a continuous domain design of the control has been carried out. Given that the majority of these converter are nowadays implemented in digital systems, a discretization of the control must be considered. This is out of the scope of this report, and will be dealt with in a different article.

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